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ABSTRACT

The mathematics curriculum in the first 2 years of college is a tool created to assist in solving problems. The current mathematics curriculum has changed little; the same topics, tied to the engineering and science curriculum, are taught as they were being taught in 1945. The problems that students need to solve have changed however. Both the business and social science curricula are bristling with problems that need mathematical tools for their solutions, while the computer science curriculum is placing enormous demands on students' mathematical problem-solving capacity. Though the current well-thought-out and well-tested math curriculum remains a marvelous course of study for students of engineering, physics, science, and mathematics, it no longer satisfies the changing needs of perhaps 75% of the students being taught. New topics and new approaches, many of which are made possible by computers, are required if students are to be well served. Because the curriculum is changing so rapidly, methods must be devised and an effective learning environment must be created to encourage the mature mathematics instructor to learn on the job. As the skills needed for problem solving change under the impact of numeric and symbolic computer computation, the focus of mathematics as a course of study in problem solving must be changed from the manipulative skills needed to solve differential equations to the conceptual skills needed to manipulate ideas; and the methods of instruction and the role of the teacher must also change. (LAL)

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Abstract

Mathematics as a Course of Study in Problem Solving: Then and Now

by
Wade Ellis, Jr.

The mathematics curriculum in the first two years of college is a tool created to assist in solving problems. The enormous computational capability provided by digital computers has increased the number of problems that can be addressed and solved using elementary mathematical concepts to include problems from the social sciences, business, and the art and science of computing. The current curriculum has not yet adjusted to the larger variety of problems that fall within the scope of mathematics that might be taught in the first two years of college.

The author compares the content and style of the mathematics curriculum at the end of World War II with its content and style now. The impact of the development of the computer on the mathematics curriculum in the first two years of college is discussed. Directions for curriculum change are suggested, and the implications of curriculum change for teacher currency, concept presentation, and curriculum content are explored.

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Mathematics as a Course of Study in Problem Solving: Then and Now

by

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Mathematics has has been a part of the college curriculum for a long, long time. Historically, from the Greeks through modern times, some knowledge of mathematics has been part of every educated person's experience. In more recent times, the usefulness of mathematics as a problem solving tool has been emphasized by the rise in technology beginning with the Industrial Revolution. This paper will focus upon this problem solving aspect of mathematics -- as it might have been at the end of World War II and as it is now.

Then

In the mid-1940's, the high-school curriculum included, with some variation, the following:

Algebra I, Algebra II and Geometry
Trigonometry and Advanced (College) Algebra
Solid Geometry

The first three year-long courses were intended as an introduction to the symbols and pictures used in mathematics. The second two courses were intended as the finishing touches for students intent upon an engineering, science or mathematics career in college. Great importance was placed upon speed and accuracy in the manipulation of symbols and upon geometric proof using pictures. The able student finished this curriculum well prepared for the college mathematics curriculum. Non-science students required no further mathematical training in college beyond the first three years of high school.

The beginning college curriculum was tied to the engineering and science curriculum and was an integral part of it. The following courses comprised that curriculum:

Calculus of one variable
Calculus of several variables
Differential Equations

These courses were intimately tied to the engineering curriculum in that problems to be solved in the engineering and physics textbooks used techniques taught in these courses. The ideas that these textbook problems were contrived to give the student some success in the problem-solving process was well-known, encouraged and applauded. In the best of circumstances, students became confident of their problem solving ability and the mathematical resources that were available to them. In less auspicious circumstances, mathematics was seen as a burden that students endured until they were allowed to use the formulas that appeared in the CRC Handbook and other scientific and engineering handbooks. If you could plug things in, then you would do all right.

Problem solving based on this curriculum was apparently successful as the group of students educated under this course of study was able to put a man on the moon, no small problem-solving feat. Calculus and differential equations, the mathematics of Newtonian physics, were the keys to the solution of that problem. The combined mathematics/physics/engineering/science curriculum was a raging success.

But times change and so do the problems needing solutions. For example, are the current problems still best approached from the point of view of Newtonian physics, or are other methods more fruitful? We no longer teach the Ptolemaic system of planetary motion because improved observations showed Ptolemy's explanation to be inadequate. Later, again with planetary motion (Mercury), Newtonian mechanics was shown to be inadequate. Newtonian physics is still a marvelous creation and is correct (and will remain so) in an enormous number of problem settings. But there are other mathematical notions that are needed to deal with physics and for the illumination of the underlying concepts of Newtonian physics the rigors of calculus may no longer be required. The differential calculus was needed to allow symbolic calculations to be substituted for numerical ones. The advent of the digital computer with its enormous numerical calculation ability might better be used to introduce Newtonian mechanics.

Quantum mechanics, which is now taught in the engineering physics courses at community colleges, requires more than calculus.

With shifts in the physics and engineering curriculum, must we still require that students learn the intimate details of differential and integral calculus and not learn and benefit from the numerical techniques that are now feasible with digital computers? Are the years of Algebra, Geometry, Trigonometry, Calculus, and Differential Equations really required or is there a short cut or a short circuit for our students?

Now

The current curriculum has changed little from the post-war years. The names of the courses are a little different and Solid Geometry has disappeared as a name and as a topic in the high-school curriculum. In colleges, a watered-down version of Linear Algebra is now taught to some sophomores. Basically, we are still teaching the same general topics to students now that we did in 1945.

The details have changed nonetheless. The emphasis on epsilon-delta proofs has increased enormously. The symbol epsilon does not appear in the 1941 revised edition of *Elements of the Differential and Integral Calculus* by Granville, Smith and Longley, and it has only two sections occupying 5 pages on the limit concept. In Leithold's 1981 fourth edition of *The Calculus with Analytic Geometry*, there is a 77 page chapter on the limit concept. Though the emphasis on epsilon-delta proofs is currently decreasing, it is far greater than it was in 1945. Function notation is more widely used than in the past in mathematics courses (but not in engineering/science courses). The quality of textbooks has improved immensely with more numerous and better examples and graphs, though we still teach mostly by lecturing with little dependence on a textbook except for problems. Unfortunately, the topics are still the same.

Have the problems we wish our students to solve changed? Oh my, yes! The business curriculum and the social science curriculum are now bristling with problems that need mathematical tools for their solutions. The exploding computer science curriculum is placing enormous demands upon the mathematical problem solving capability of students we teach.

The business curriculum appears to require strong data analysis problem-solving methods including graphically displayed data analysis techniques. This places a premium on understanding concepts of data analysis and diminishes the need for strong manipulative skills in algebra.

Statistics is important and so are the ideas of increasing and decreasing functions, but from a graphical point of view. Linear programming is important as well but the analysis of the results of the calculations is much more important than the calculations themself. The same needs are faced in the social science curriculum.

The computer science curriculum, on the other hand, places much more emphasis on manipulative skills in dealing with finite state machines, function notation and proofs of an inductive nature. In addition, new topics such as semigroups, graph theory, algorithms, and combinatorics are part of the curriculum. In addition, a formal programming language is needed to concisely represent and understand algorithms.

As an example of the new pressure placed upon the mathematics curriculum, let us consider some topics from the table of contents of a recent text on mathematical structures for computer science. We find topics such as:

Induction and Recursion
Relations and Digraphs
Connectivity and Warshall's Algorithm
Posets
Representations of Special Grammars
Undirected Trees
Products and Quotients of Groups
Finite-State Machines
Coding of Binary Information and Error Detection

These topics are of special interest to us because they are from a sophomore textbook, written by mathematicians, to be taught by mathematicians. It is not a list of topics from an isolated lecture-note type presentation of an experimental nature, but rather a course that has been taught for several years at Drexel University. In fact, the Computer and Information Science Department at my community college is asking the Mathematics Department to teach a course like it this coming year.

The Impact of Computers

The mathematics community has a problem. We have an extremely well-thought-out and tested curriculum in many versions (The Ohio State University teaches 9 different varieties of Beginning Calculus). It does not

satisfy the changing needs of perhaps 75% of the students we teach although it is still a marvelous course of study for students of engineering, physics, science, and mathematics. The computer has in large part brought on this problem in two ways. First, the availability of fast and accurate computational devices has increased the accessibility of problems whose solutions use established mathematical techniques but require massive computational capability (if done by hand or with a mechanical desktop calculator). The use of computers to solve such problems has increased the effectiveness of investigations in the social science and business disciplines and thereby changed the curriculum needs of the students in these disciplines. Second, the complexity and development of computers and computing has required the application of mathematical ideas not contained in the "old" mathematics curriculum to problems in computer science. In some cases, computer science problems have required the creation of new mathematical ideas for their solution. These new ideas will become candidates for inclusion in the mathematics curriculum.

Microcomputers have also given us a new tool to use in presenting information and ideas to students. The availability of a microcomputer (with a suitably large screen display) can greatly enhance the presentation of material in class through rapid calculation and accurate graphs. The microcomputer can also be used to present information in a highly structured manner to individual students in laboratory settings. Microcomputers can also provide exercises in discovery learning of mathematical concepts.

Finally, the availability of symbolic manipulation programs on microcomputers reduces the need for computational skill with symbols in much the same way the hand-calculator reduced the need for computational skill development in arithmetic. This new availability calls into question much of the course of study in high schools and colleges and is both a blessing and a curse. Textbook writers and manufacturers have a vested interest in the current curriculum. In our bottom line society, they are trend setters. It will be difficult to change their minds about what is to be in the curriculum. We, as teachers, also have a vested interest in this curriculum since we have learned over many years how to deal effectively with it: how to present it; how to measure student success in it; and how to remediate student difficulties with it. So much for the curse. The blessing is that the reduction in the need for computational skill development throughtout the curriculum provides us with the needed time

to introduce the new topics and approaches that will be required if we serve our students well.

The Computer in Matheamtics

For those of us who believe that computers like calculators are going to go away, let me say that mathematical research is driven in large part by examples, and examples often require calculations. The more complex these calculations can be, the more quickly patterns can be discerned and tested. Computers can help and are helping in doing these calculations, both numerically and symbolically. They are here to stay.

Teacher Preparation

The content, presentation styles, and methods of mathematics are changing rapidly. As a teacher in the field, I feel woefully underprepared to teach the new topics with the newly available teaching techniques, but am extremely excited about the possibilities. I do not know enough, although I have taken advanced graduate level courses in combinatorics, real and complex analysis, number theory, and abstract algebra. I have no idea what Warshall's Algorithm is and no guess as to how to teach students about it.

I have spent an entire sabbatical year studying the uses of computers in teaching and have taught a course entitled "The Microcomputer as a Research Tool in Mathematics." I have written textbooks about the computer language BASIC. Still, my hands begin to shake when I attempt to present mathematical ideas using a computer in a classroom. There is so much that can go wrong or right that I know nothing about. Students are frequently asking easy questions that I have no idea how to answer.

We can teach the current crop of novice teachers to know more of the content and we can teach them to use the current new techniques. But how much impact will these new teachers have? For that matter, when is the last time we saw someone who was embarking upon a career in teaching mathematics?

We are currently going through the hiring process for two positions at West Valley College. All of the applicants completed their training and coursework 3 to 20 years ago. Only one was completing his coursework this year and this applicant was a retrainee from another discipline. Many

of the applicants had Ph.D.'s from other disciplines vaguely related to mathematics. The message, for me at least, is what we have is what we are going to have for the forseeable future. We have seen the enemy and it is us.

Because the curriculum is changing so rapidly, we must devise methods of encouraging the mature mathematics instructor to learn on the job. Since nobody, in my very humble opinion, knows where any of this is going, we must encourage ourselves to delve deeply into the new situation in mathematics each and every day. This means that we must seek reduced teaching loads so that we may have the time to learn the new material and experiment with the new techniques. In my opinion, workshops will not work because they give the false impression that there is the possibility of a quick fix for these problems. Each department must form itself into an effective learning environment for the new mathematics that most of us do not know. We must create an experimental learning environment with ample access in our offices and homes to the new computer technology so that we may internalize what it is we teach in the same way that we have internalized the limit processes of the calculus.

The Future

As the skills needed for problem solving change under the impact of computer computation (both numeric and symbolic), we must change the focus of *mathematics as a course of study in problem solving* from the manipulative skills required to solve differential equations to the conceptual skills needed to manipulate ideas. Thus, in the future, we must concentrate on modes of thinking and not on skills in manipulation. We should concentrate on five thinking skills:

Axiomatic Thinking (Theorem/Proof)
Programmatic Thinking (Large Problems into Small)
Algorithmic Thinking (Finite Solution Processes)
Probabilistic Thinking (Statistical Approaches)
Eclectic Thinking (Are there several ways to solve a problem?)

If we are to restructure the curriculum, then we need to consider those concepts and topics which introduce, develop and enhance these patterns of thought. The use of these mathematical modes of thought in

problem solving is the direction in which I believe the first two years of college mathematics should go.

In addition, we should concentrate on preparing our students to read and understand mathematical ideas of all kinds and then to use these newly acquired ideas in solving problems. Along with this, we need to encourage our students to write and speak mathematics well so that as they acquire new knowledge they can pass it on to others. I hope in this endeavor we are able to encourage our students to spell it all out rather than use the terse and elegant journal style we all admire so much.

If we begin to change the curriculum in these directions, then we will find that the current lecture form of instruction will not suffice. We will need to break away from the teacher as all-knowing knowledge provider and begin to think of the teacher as guide and advisor in a student's quest for knowledge. We will begin also to depend more upon guided discovery learning techniques and written mathematical materials. Perhaps, here again we will call upon the computer to assist us in structuring learning environments that will gently push the students toward improving their powers as problem solvers using mathematical modes of thought.

The future is bright with challenges. Let us hope we can meet them.

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